IB Probability Practice

1. You have two coins. Coin A is fair – it gives a 50% probability of heads or tails. Coin B is loaded – it has a 60% chance of giving tails.

a) If you flip each coin 4 times, what is the probability each will give you no heads?

b) Suppose the coins look the same and you mix them up. You pick one, flip it four times, and you get tails every time. What is the probability you've been flipping Coin B?

c) How many times would you have to flip the coin, and get all tails, until you were 99% sure you had coin B?

Hint: What is the probability of getting n *consecutive tails on each coin?*
And then what is the % chance you are flipping coin B if you've already gotten that result?

2. Given $P(B|A) = 0.4$ and $P(B \cap A) = 0.3$, what is $P(A')$?

3. How many ways are there to arrange:

a) 4 boys and 3 girls, in a line, so that no two boys are beside each other?

b) 4 boys and 3 girls, in a line, so that no two girls are beside each other?

c) 4 boys and 3 girls, in a line, where all four boys aren't together consecutively?

d) The letters of MATHEMATICS so that no two vowels are together?

e) The letters of MATHEMATICS so that all the vowels are together?

Answers

1. You have two coins. Coin A is fair – it gives a 50% probability of heads or tails. Coin B is loaded – it has a 60% chance of giving tails.

a) If you flip each coin 4 times, what is the probability each will give you no heads?

Coin A: P(4 tails) = $(1/2)^4 = 1/16$
Coin B: P(4 tails) = $(3/5)^4 = 81/625$

b) Suppose the coins look the same and you mix them up. You pick one, flip it four times, and you get tails every time. What is the probability you've been flipping Coin B?

This is Bayes' Theorem. We already know what the absolute probability of getting this outcome is for each coin ... so we can use those to figure out the relative probability of tje coin having been B.

P(It is Coin B) = (81/625) / (81/625 + 1/16) = 0.6746

c) How many times would you have to flip the coin, and get all tails, until you were 99% sure you had coin B?

Probability of Coin A giving all tails after *n* flips: $(1/2)^n$
Probability of Coin B giving all tails after *n* flips: $(3/5)^n$

P(it is coin B after getting *n* tails) = $(3/5)^n$ / [$(3/5)^n + (1/2)^n$] <-- This is BAYES
And we want this to be more than 99%:

So we need to solve: $$\frac{\left(\frac{3}{5}\right)^n}{\left(\frac{3}{5}\right)^n+\left(\frac{1}{2}\right)^n}>0.99$$

This might actually be a little tough. I'm going to take the reciprocal: $$\frac{\left(\frac{3}{5}\right)^n+\left(\frac{1}{2}\right)^n}{\left(\frac{3}{5}\right)^n}<\frac{100}{99}$$

So that I can simplify the fraction: $$1+\frac{\left(\frac{1}{2}\right)^n}{\left(\frac{3}{5}\right)^n}<\frac{100}{99}$$

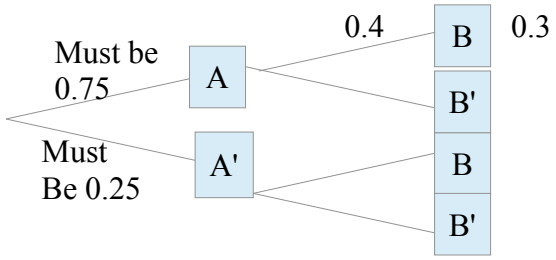
Subtract 1 from both sides and simplify the fractions: $$\left(\frac{1/2}{3/5}\right)^n<\frac{1}{99}$$
$$\left(\frac{5}{6}\right)^n<\frac{1}{99}$$

And calculate: $$n\log(5/6)<\log(1/99)$$

$$n(-0.07918) < -1.9956$$
$$n > \frac{-1.9956}{-0.07918}$$
$$n > 25.2$$

So you would need 26 consecutive tails.

2. Given P(B|A) = 0.4 and P(B∩A) = 0.3, what is P(A')?



3. How many ways are there to arrange:

a) 4 boys and 3 girls, in a line, so that no two boys are beside each other?

B G B G B G B <-- only possible arrangement
(Arrangements of boys)x(Arrangements of girls) = 4!×3! = 144

b) 4 boys and 3 girls, in a line, so that no two girls are beside each other?

Group two girls together into one unit ... there are 6!×2! arrangements that have two girls together.
Total arrangements: 7!
Arrangements *without* two girls together: 7! - 6!2! = 3600

c) 4 boys and 3 girls, in a line, where all four boys aren't together consecutively?

Arrangements with all the boys together: (arrangements of boys)×(arrangements of 3 girls + boy unit) = 4!×4! = 576

Arrangements without all the boys together: 7! - 576 = 4,464

d) The letters of MATHEMATICS so that all the vowels are together?

Ways to arrange the group of four vowels: 4!/2! = 12
Ways to arrange the remaining 7 consonants (two doubles) and the unit of four vowels:
8! / (2!2!) = 10,080

e) The letters of MATHEMATICS so that no two vowels are together?

Arrange the consonants: 7! / 2! 2! = 1260

This gives us C C C C C C C. We need to pick four of the 'slots' that vowels could go into.
Slots are: XCXCXCXCXCXCXCX (there are 8). This represents 8C4 / 2! options = 35 options

So we have 1260 × 35 = 44,100