Continuous Distributions (MDM4U Chapter 7)

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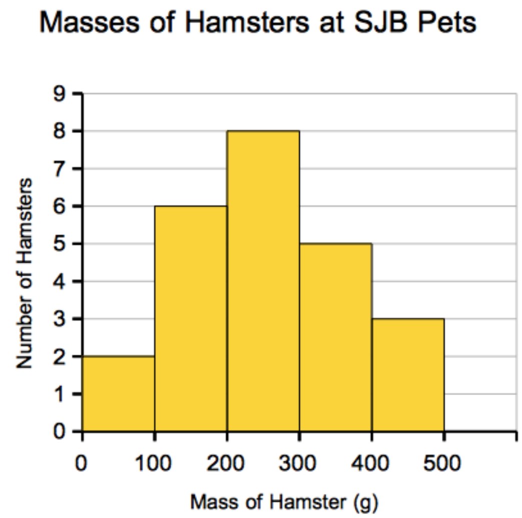
1. Consider the histogram on the right.



a) Is this a uniform distribution?

b) What is the median hamster mass?

c) What is the probability that a given hamster weighs less than 200 g?

d) What is the probability that a given hamster weights more than 350 grams?

2. The mass of each beet from a harvest in August 2017 follows a normal distribution with $\mu = 120.0$ g and $\sigma = 14.0$ g

a) What is the probability of any particular beet having a mass less than 100 g?

b) What is the probability of any particular beet having a mass or more than 150 g?

c) Beets that weigh less than 100 g are called *rejects* and beets that weight more than 150 g are called *mutants*. Both are made into beet juice. From a truckload of 240 000 beets, how many are going to be juiced?

3. Consider the same harvest of beets: $\mu = 120.0$ g and $\sigma = 14.0$ g.

a) If you took a bunch of samples of 20 beets each, what would you expect the standard deviation of the means to be?

b) If you took a bunch of samples of 50 beets each, what would you expect the standard deviation of the means to be?

c) Why does the "standard deviation of the means" shrink as the sample size increases?

4. The probability that a 504 (King Street) Streetcar would show up on time in August 2017 was 60%.

a) For a regular weekday of 100 streetcars, how many do you expect to be on time?

b) Using the binomial distribution, what is the probability that *exactly* 60 would be on time?

c) Using the binomial approximation, what is the probability that *exactly* 60 would be on time?

5. A survey of Toronto residents finds that 38% would vote for John Tory to be mayor again. The survey is considered accurate to ±4%, 19 times out of 20, then:

a) What is the:

i) confidence interval?

ii) confidence level?

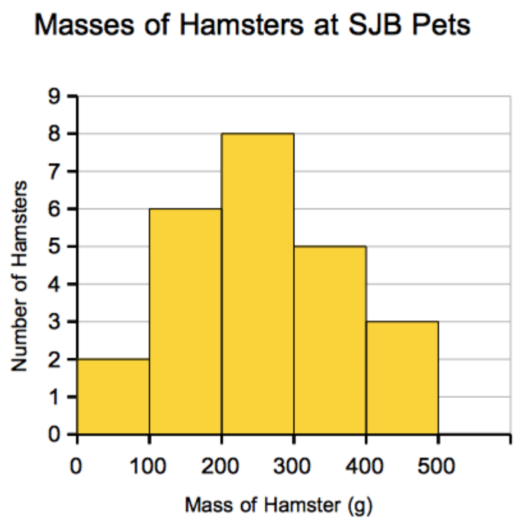
iii) margin of error?

b) How many people were polled?

c) What would the margin of error be at a 99% confidence level?

Answers

1. Consider the histogram on the right.



a) Is this a uniform distribution?
No. There are different numbers (probabilities) of hamsters in different intervals. There would have to be the same number in each interval to be uniform.

b) What is the median hamster mass?
There are 24 hamsters, so the 12th hamster is the "middlest". He is in the 200<x<300 interval and so the median is 250 g.

c) What is the probability that a given hamster weighs less than 200 g?
8 of the 24 hamsters weight less than 200, so the probability is 8/24 = 1/3 = **33%**

d) What is the probability that a given hamster weights more than 350 grams?
We can assume half of the 300<x<400 hamsters (five) are above 350 g, so that's 2.5 hamsters. Plus the 3 full hamsters from the 400<x<500 interval, makes about 5.5 hamsters.

5.5 out of 24 = 11/48 = **22.9%**

2. The mass of each beet from a harvest in August 2017 follows a normal distribution with $\mu = 120.0$ g and $\sigma = 14.0$ g

a) What is the probability of any particular beet having a mass less than 100 g?

$$z=\frac{x-\mu}{\sigma}=\frac{(100-120)}{14}=\frac{20}{14}=-1.43$$

$$P(z<-1.43)=0.0764=7.64\%$$

b) What is the probability of any particular beet having a mass or more than 150 g?

$$z=\frac{x-\mu}{\sigma}=\frac{150-120}{14}=\frac{30}{14}=+2.14$$

$$P(z>2.14)=1-P(z<2.14)=1-0.9838=0.0162=1.62\%$$

c) Beets that weigh less than 100 g are called *rejects* and beets that weight more than 150 g are called *mutants*. Both are made into beet juice. From a truckload of 240 000 beets, how many are going to be juiced?

$$100\%-7.64\%-1.62\%=90.74\%$$

90.74% of 240 000 = 0.9074 × 240 000 = **217,776 beets**

3. Consider the same harvest of beets: μ = 120.0 g and σ = 14.0 g.

a) If you took a bunch of samples of 20 beets each, what would you expect the standard deviation of the means to be?

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{14.0\,g}{\sqrt{20}} = 3.13\,g$$

b) If you took a bunch of samples of 50 beets each, what would you expect the standard deviation of the means to be?

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{14.0\,g}{\sqrt{50}} = 1.98\,g$$

c) Why does the "standard deviation of the means" shrink as the sample size increases?

- Larger sample sizes are more likely to be close to the mean, because each outlier has less of an effect.
- Put another way, the larger the sample, the more representative of the population it might be.
- Therefore, sample means of bigger samples are closer to the actual mean of the population.
- And so the standard deviation of the sample means will be smaller.

4. The probability that a 504 (King Street) Streetcar would show up on time in August 2017 was 60%.

a) For a regular weekday of 100 streetcars, how many do you expect to be on time?

60% of 100 is **60 streetcars.**

b) Using the binomial distribution, what is the probability that *exactly* 60 would be on time?

$${}_{100}C_{60}(0.60)^{60}(0.40)^{40} = 0.08122 = 8.12\%$$

c) Using the binomial approximation, what is the probability that *exactly* 60 would be on time?

- We need the area of the normal curve from 59.5 to 60.5
- $\mu = np = 100(0.60) = 60$
- $\sigma = \sqrt{npq} = \sqrt{100(0.60)(0.40)} = 4.90$
- $z = \frac{x-\mu}{\sigma} = \frac{60.5-60}{4.90} = +0.10$ and $P(z<0.10) = 0.5398 = 53.98\%$
- $z = \frac{x-\mu}{\sigma} = \frac{59.5-60}{4.90} = -0.10$ and $P(z<-0.10) = 0.4602 = 46.02\%$
- $P(-0.10<z<0.10) = 53.98\% - 46.02\% = 7.96\%$

5. A survey of Toronto residents finds that 38% would vote for John Tory to be mayor again. The survey is considered accurate to ±4%, 19 times out of 20, then:

a) What is the:

i) confidence interval? 34% to 42%

ii) confidence level? 19/20 = 95%

iii) margin of error? 4%

b) How many people were polled?

$$E = z\sqrt{\frac{p(1-p)}{n}} \quad \text{... and so ...} \quad 0.04 = 1.960\sqrt{\frac{0.38(1-0.38)}{n}}$$

$$0.0204 = \sqrt{\frac{0.2356}{n}}$$

$$0.00041649 = \frac{0.2356}{n}$$

$$n = \frac{0.2356}{0.00041649} = 565.6 \approx 566 \quad \text{people.}$$

c) What would the margin of error be at a 99% confidence level?

$$E = (2.576)\sqrt{\frac{0.2356}{566}} = (2.576)(0.0204) = 0.0536 \approx 5\%$$