

1. Expand (Distribute) and simplify.

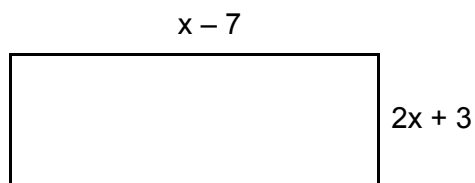
a) $(x + 4)(3x - 1)$

b) $4(x + 2)(5x - 6)$

c) $(x + 1)(x - 4) - 2x(x + 3)$

d) $(4x - 3)^2 - (4x + 3)^2$

2. Someone paints a rectangular sign of dimensions shown below.
Determine an expression (in standard form) the **area** of such a sign.



3. Factor fully.

a) $x^2 - 8x + 12$

b) $3x^2 + 10x + 7$

c) $9x^2 + 36xy + 16y^2$

d) $9x^2 + 24xy + 16y^2$

e) $3x^2 + 3x + x + 1$

f) $x^2 - 4$

g) $9x^2 - 49$

h) $121x^2 - 64y^2$

i) $x(x + 2) - 2(x + 2)$

j) $4x + 8z + 80xy$

k) $7x^2y - 21x^3y^2 + 28xy^5$

l) $x^4 - 81$

m) $2x^2 + 9x + 4$

n) $2x^2 + 11x + 12$

o) $12x^2 + 23x - 2$

p) $6a^2 + 1a - 26$

q) $16x^2 + 40xy + 25y^2$

r) $256x^8 - 6561$

s) $-16x^2 + 81$

t) $3x^2 - 75$

u) $48x^4 - 216x^2 + 243$

4. The area of a rectangular field is given by the expression;

$$A = 36x^2 - 114x + 70$$

Determine expressions for the side lengths of this field.

5. Fully factor $V = 7x^2 - 28x - 84$ to get expressions for the length/width/height of a rectangular prism.

6. Find the length/width/height of a rectangular prism with the volume given by:
 $V = 4x^3 - 12x^2 - 72x$

7. A toy rocket shot from a point A to the left of a building, flies over it and lands on the other side.
The height h (in feet) of the rocket depends on the horizontal distance d (in feet) according to the equation:

$$h = -0.5d^2 + 2d + 10.5$$

a) Factor the equation above.

b) Using the answer from part a) above (or otherwise), sketch the path of the rocket

8. A test missile shot from one side of a mountain; it flies over the mountain and lands on the other side.
The height h (in metres) of the missile depends on the time t (in seconds) according to the equation

$$h = -5t^2 + 40t - 60$$

a) Factor the equation above.

b) Using the answer from part a) above (or otherwise), sketch the path of the missile.

Answers

1. Expand (Distribute) and simplify.

$$\begin{aligned} \text{a)} \quad & (x + 4)(3x - 1) \\ & = 3x^2 - 1x + 12x - 4 \\ & = \mathbf{3x^2 + 11x - 4} \end{aligned}$$

This is "FOIL"ing
and collecting like terms

$$\begin{aligned} \text{b)} \quad & 4(x + 2)(5x - 6) \\ & = (4x+8)(5x-6) \\ & = 20x^2 - 24x + 40x - 48 \\ & = \mathbf{20x^2 + 16x - 48} \end{aligned}$$

multiply the 4 into the first bracket only
then FOIL

$$\begin{aligned} \text{c)} \quad & (x + 1)(x - 4) - 2x(x + 3) \\ & = (x^2 - 4x + x - 4) + (-2x^2 - 6x) \\ & = x^2 - 4x + x - 4 - 2x^2 - 6x \\ & = \mathbf{-x^2 - 9x - 4} \end{aligned}$$

I multiplied the -2x into the bracket; that's why it became +

$$\begin{aligned} \text{d)} \quad & (4x - 3)^2 - (4x + 3)^2 \\ & = (4x-3)(4x-3) - (4x+3)(4x+3) \\ & = 16x^2 - 12x - 12x + 9 - (16x^2 + 12x + 12x + 9) \\ & = 16x^2 - 12x - 12x + 9 - 16x^2 - 12x - 12x - 9 \\ & = 16x^2 - 12x - 12x + 9 - 16x^2 - 12x - 12x - 9 \\ & = \mathbf{-48x} \end{aligned}$$

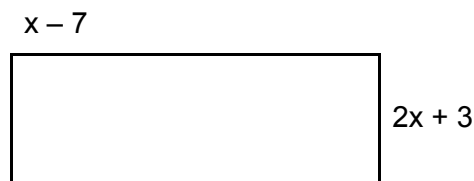
Lots of stuff cancels here.

2. Someone paints a rectangular sign of dimensions shown below.

Determine an expression (**in the standard form**) the area of such a sign.

^ this means multiply it out; no brackets allowed

Area of a rectangle = Length \times Width



A = length \times width

$$A = (x-7)(2x+3)$$

$$A = 2x^2 + 3x - 14x - 21$$

$$\mathbf{A = 2x^2 - 11x - 21}$$

3. Factor fully.

$$\begin{aligned} \text{a)} \quad & x^2 - 8x + 12 \\ & = \mathbf{(x-6)(x-2)} \end{aligned}$$

← two numbers that multiply to +12 and add to -8
(Shortcut available, nothing in front of x^2)

$$\begin{aligned} \text{b)} \quad & 3x^2 + 10x + 7 \\ & = \mathbf{3x^2 + 3x + 7x + 7} \end{aligned}$$

← two number that multiply to +21 and add to +10
(no shortcut available; use decomposition)

- c) $9x^2 + 36xy + 16y^2$ ← two numbers that multiply to +144 and add to +36
(does not exist)
Not factorable
- d) $9x^2 + 24xy + 16y^2$ ← two numbers that multiply to +144 and add to +24
 $= 9x^2 + 12xy + 12xy + 16y^2$
 $= \underline{3x}(3x+4y) + \underline{4y}(3x+4y)$
 $= (3x+4y)\underline{(3x+4y)}$
 $= (3x+4y)^2$ ← There was a shortcut available here.
 Called "perfect square trinomial"
 square root of $9x^2 \rightarrow 3x$
 square root of $16y^2 \rightarrow 4y$
 middle term: $2(3x)(4y) = 24xy$
- e) $3x^2 + 3x + x + 1$ ← Four terms? Use **grouping**
 $= 3x(x+1) + (x+1)$
 $= (3x+1)(x+1)$
- f) $x^2 - 4$ ← difference of squares
 $= (x-2)(x+2)$
- g) $9x^2 - 49$
 square root of $9x^2 \rightarrow 3x$
 square root of $49 \rightarrow 7$
 Separated by a - sign? Difference of Squares!
 $= (3x-7)(3x+7)$
- h) $121x^2 - 64y^2$
 square root of $121x^2 \rightarrow 11x$
 square root of $64y^2 \rightarrow 8y$
 separated by a - → Difference of Squares
 $= (11x-8y)(11x+8y)$
- i) $x(x+2) - 2(x+2)$ ← Also grouping, but some was already done for you
 $= (x+2)(x-2)$
- j) $4x + 8z + 80xy$ ← Common factoring ... what's the biggest number
 $= 4(x + z + 20xy)$ that divides nicely into all of these? It is **4**
- k) $7x^2y - 21x^3y^2 + 28xy^5$
 $= 7xy(x - 3x^2y + 4y^4)$ Common factoring again.
- l) $x^4 - 81$ ← difference of squares
 $= (x^2-9)(x^2+9)$ ← difference of squares AGAIN
 $= (x+3)(x-3)(x^2+9)$ * The + version can not be factored

$$\begin{aligned}
 \text{m)} \quad & 2x^2 + 9x + 4 \\
 & = 2x^2 + 8x + x + 4 \\
 & = 2x(x+4) + (x+4) \\
 & = \mathbf{(2x+1)(x+4)}
 \end{aligned}$$

$$\begin{aligned}
 \text{n)} \quad & 2x^2 + 11x + 12 \\
 & = 2x^2 + 8x + 3x + 12 \\
 & = 2x(x+4) + 3(x+4) \\
 & = \mathbf{(2x+3)(x+4)}
 \end{aligned}$$

$$\begin{aligned}
 \text{o)} \quad & 12x^2 + 23x - 2 \\
 & = 12x^2 + 24x - x - 2 \\
 & = 12x(x+2) - (x+2) \\
 & = \mathbf{(12x-1)(x+2)}
 \end{aligned}$$

$$\begin{aligned}
 \text{p)} \quad & 6a^2 + 1a - 26 \\
 & = 6a^2 + 13a - 12a - 26 \\
 & = a(6a+13) - 2(6a+13) \\
 & = \mathbf{(6a+13)(a-2)}
 \end{aligned}$$

$$\begin{aligned}
 \text{q)} \quad & 16x^2 + 40xy + 25y^2 \\
 & = 16x^2 + 20xy + 20xy + 25y^2 \\
 & = 4x(4x+5y) + 5y(4x+5y) \\
 & = \mathbf{(4x+5y)(4x+5y)}
 \end{aligned}$$

$$\begin{aligned}
 \text{r)} \quad & 256x^8 - 6561 && \leftarrow \text{difference of squares} \\
 & = (16x^4 - 81)(16x^4 + 81) && \leftarrow \text{difference of squares again} \\
 & = (4x^2-9)(4x^2+9)(16x^4+81) && \leftarrow \text{difference of squares again} \\
 & = \mathbf{(2x-3)(2x+3)(4x^2+9)(16x^4+81)} && \leftarrow \text{the rest aren't factorable}
 \end{aligned}$$

$$\begin{aligned}
 \text{s)} \quad & -16x^2 + 81 \\
 & = -(16x^2 - 81) \\
 & = \mathbf{-(4x-9)(4x+9)}
 \end{aligned}$$

$$\begin{aligned}
 \text{t)} \quad & 3x^2 - 75 \\
 & = 3(x^2 - 25) && \leftarrow \text{"Common Factor" out a 3} \\
 & = \mathbf{3(x-5)(x+5)} && \leftarrow \text{Then what's left is a "difference of squares".}
 \end{aligned}$$

4. The area of a rectangular field is given by the expression;

$$A = 36x^2 - 114x + 70$$

Determine expressions for the side lengths of this field.

**** Strategy: FACTOR.** Two numbers that multiply to $(36)(70) = +2520$
and add to -114
THEY ARE -30 and -84

$$A = 36x^2 - 30x - 84x + 70$$

$$A = 6x(6x-5) - 14(6x-5)$$

$$A = (6x-5)(6x-14)$$

\rightarrow actually fully factors to $2(6x-5)(3x-7)$

$$A = (\text{length})(\text{width})$$

The length could be $6x-5$

The width could be $6x-14$

5. Fully factor $V = 7x^2 - 28x - 84$ to get expressions for the length/width/height of a rectangular prism.

$$V = (\text{length})(\text{width})(\text{height})$$

take out a common factor

$$V = 7x^2 - 28x - 84$$

$$V = 7(x^2 - 4x - 12)$$

$$V = 7(x-6)(x+2)$$

The length could be **$x+2$**

The width could be **$x-6$**

The height could be **7**

6. Find the length/width/height of a rectangular prism with the volume given by:
 $V = 4x^3 - 12x^2 - 72x$

$$V = 4x(x^2 - 3x - 18)$$

$$V = 4x(x-6)(x+3)$$

The length could be $4x$

The width could be $x-6$

The height could be $x+3$

7. A toy rocket shot from a point A to the left of a building, flies over it and lands on the other side.
The height h (in feet) of the rocket depends on the horizontal distance d (in feet) according to the equation:

$$h = -0.5d^2 + 2d + 10.5$$

- a) Factor the equation above.

**** take out the -0.5 first ****

$$h = -0.5(d^2 - 4d - 21)$$

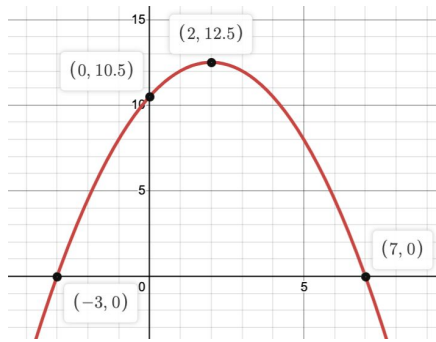
$$h = -0.5(d - 7)(d + 3)$$

- b) Using the answer from part a) above (or otherwise), sketch the path of the rocket

**** be able to find vertex ****

d-intercepts: $+7$ and -3

Axis of symmetry: $d = \frac{7+(-3)}{2} = \frac{4}{2} = 2 \rightarrow$ plug into equation to find h .



8. A test missile shot from one side of a mountain; it flies over the mountain and lands on the other side. The height h (in metres) of the missile depends on the time t (in seconds) according to the equation;

$$h = -5t^2 + 40t - 60$$

- a) Factor the equation above. $h = -5(t^2 - 8t + 12)$
 $h = -5(t-2)(t-6)$

- b) Using the answer from part a) above (or otherwise), sketch the path of the missile.

