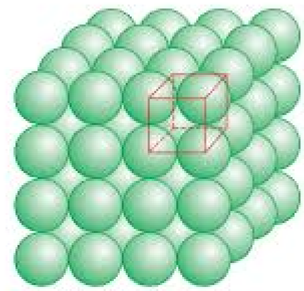
MTH1W

Volume (Challenge Problems)

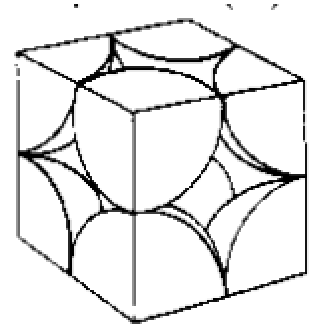
This is the most inefficient way to stack spheres →



It leaves a lot of empty space.

If we cut the balls of a few layers across the centers (see box drawn on right), we'd get a cube that could be **tessellated** (repeated) over and over to 'regenerate' the stack of balls.

This cube is called a **unit cell** in chemistry.



1. If the radius of a sphere is 200 cm:

a) How long is the side length of this cube?

b) What is the volume of the cube?

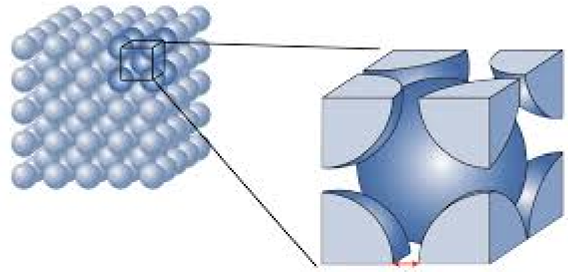
c) What volume is *actually occupied* by spheres?

d) The **packing efficiency** of the cube is the percentage of the cube that is actually taken up by spheres. Higher efficiency = Less unoccupied ('wasted') space.

What is the packing efficiency of this particular stack of spheres?

e) Does packing efficiency depend on the radius of the sphere?

This is the medium-efficient way to stack spheres →

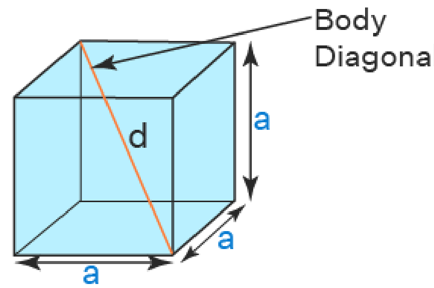


The unit cell is also shown.

The **body diagonal** is 4 radii in this unit cell.

2. If the radius of a sphere is 200 cm:

a) How long is the body diagonal?



b) The Pythagorean theorem in 3 dimensions is:
$d^2 = a^2 + b^2 + c^2$
Using that, how long is the side length of this unit cell?

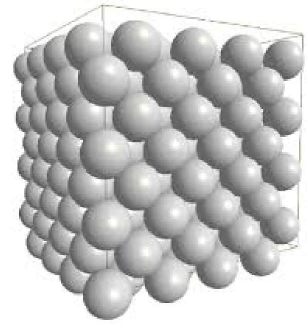
c) What is the volume of the unit cell?

c) What volume is *actually occupied* by spheres?

d) What is the packing efficiency of this particular stack of spheres?

e) Does packing efficiency depend on the radius of the sphere?

This is the most-efficient way to stack spheres →

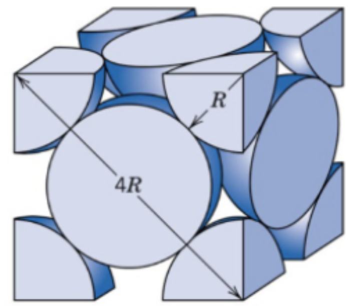


The unit cell is also shown.

The **face diagonal** is 4 radii in this unit cell.

3. If the radius of a sphere is 200 cm:

a) How long is the side length of this unit cell?



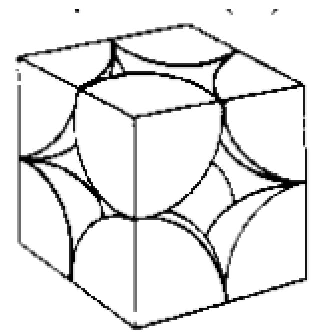
b) What is the volume of the unit cell?

c) What volume is *actually occupied* by spheres?

d) What is the packing efficiency of this particular stack of spheres?

e) Does packing efficiency depend on the radius of the sphere?

4. Let's revisit the least efficient cubic packing:



Note that there's a big chunk of empty space in between these eighths.

What is the largest sphere that could fit inside that hole?

5. What is the packing efficiency for this basketball in a glass case?

6. What is the packing efficiency for these pipes (diameter 1 m)?
Does the packing efficiency depend on the diameter?
Does the packing efficiency depend on the length of the pipe?